**Limits of a Function**

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The limit of a function is when is close to .

Mathematically,

( tends to )

If , .

,

,

,

Since left-hand limit right-hand limit, a limit exists.

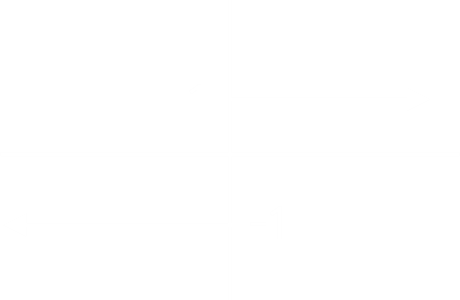
## Limit of a Function and Limit Laws

2-sided limit

1-sided limit (Right Hand Limit)

1-sided limit (Left Hand Limit)

If RHL LHL, then limit exists.



Limit does not exist.

Infinite Limits:

Limit does not exist.

Limit exists.

### Limit Laws

if

### Limits of Polynomial Functions

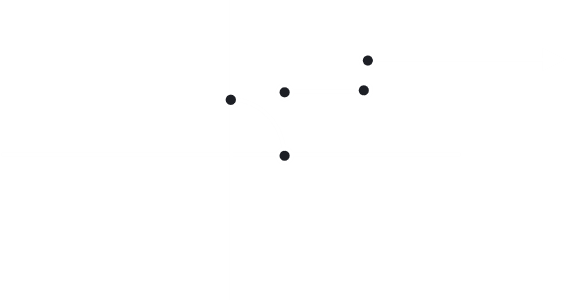
### Limits of Rational Functions

using closer values gives larger result)

L.H.L. R.H.L. so limit does not exist.

### Limits of Piecewise Functions

Limit exists.



Limit does not exist.

Limit does not exist.

### Trigonometric Limits

## Squeezing Theorem

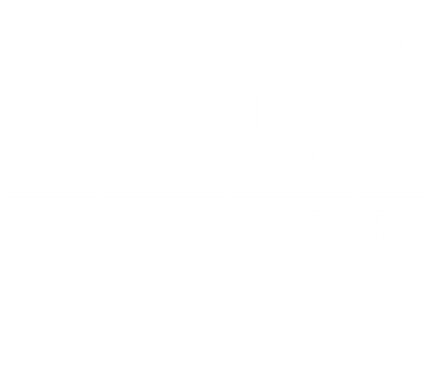
Suppose , and are functions such that for all values of in the open interval that contains a number , except possible at itself.

If and , then

Example:

Show that , using the squeezing theorem.

Let a unit circle have radius and centre .



Area Area Sector < Area

and

Find the limit of:

Let .

Limit does not exist since .

## Continuity

If a function has no breaks, it is called a continuous function.

* Function must be defined.
* Limit must exist.
* Limiting value must be equal to the functional value.

Case A:

Not continuous since makes function undefined.

Case B:

is defined at , .

and so limit exists.

limit value, so not a continuous function.

Case C:

is defined at , .

and so limit exists.

limit value.

This is a continuous function.

If is a vertical asymptote is said to have infinite discontinuity at (Case A).

If , is said to have finite or jump discontinuity at .

If , is said to have removable discontinuity (Case B).

Example:

Determine if is continuous at .

is defined.

is not continuous at . It has removable discontinuity.

Polynomial functions are continuous everywhere. Rational functions are continuous everywhere except where the denominator .

Find at what points the functions are continuous:

1. discontinuous at

continuous

1. discontinuous at and
2. discontinuous at
3. continuous
4. continuous

discontinuous at

Show that is continuous everywhere.

and are polynomial functions, so these are continuous. However, there is a possible discontinuity at

The function is continuous everywhere.

Using limits, find the horizontal asymptote of

If ,

( and )

and a horizontal asymptote exists.

Let .

Use the Sandwich theorem to find the horizontal asymptote of

(by Sandwich theorem)

Now,